MAT1A01

Numbers, Inequalities and Absolute Values

Dr Craig

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Introduction

Who:
Dr Craig

What:
Lecturer and course co-ordinator for MAT1A01

Where:
C-Ring 508

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Course information

Lecture times:
Tuesday 8:50–10:25 and Wednesday 17:10–18:45
Venues:
C-Les 102, C-Les 103, C-Les 401, C-Les 402

Tutorials: Tuesday afternoons
Either 13:50–15:25 *or* 15:30–17:05
There **will** be a tutorial this afternooon.

First session:
C-Les 204, C-Les 404, D-Les 202, D-Les 104
Second session:
C-Les 203, C-Les 204, D-Les 202, D-Les 104

Textbook

Calculus: Early Transcendentals (7th edition) by
James Stewart
Metric International Version, Thomson Brooks/Cole
ISBN nr: 978-0-538-49887-6

Older editions are *fine*. Some of the exercises are different, but the material is mostly the same.

You will use this textbook again for 2nd year Calculus, so if your course requires you to do at least two years of maths, this is a good investment. There are also free online books listed in the Learning Guide.

Important Dates

Monday 23 February 6pm – Electronic Test

Saturday 7 March – Semester Test 1

Monday 11 May – Semester Test 2

Saturday Classes

Students whose Gr12 mark was less than 65%, or whose NBT score was less than 30, are **strongly advised** to attend Saturday morning classes. These will take place from 9am to 12pm on Saturday mornings. The venue will be announced later in the week.

Students who obtain less than 40% for Semester Test 1 will have to attend **compulsory** Saturday morning classes.

$\mathbb{Q}$uestions?

Some of the topics in MAT1A01

- Complex numbers
- Propositional logic
- First-order logic
- Proof techniques
- Limits
- Derivatives of trigonometric functions
- Proofs of differentiation rules
- The chain rule
- Anti-derivatives
- Integration - area under curves

Number systems

Some numbers cannot be written as $\frac{m}{n}$ for $m, n \in \mathbb{Z}$. These are called *irrational* numbers. For example:

$$\sqrt{2} \qquad \sqrt[3]{9} \qquad \pi \qquad e \qquad \log_{10} 2$$

The set of all rational and irrational numbers is known as the set of *real numbers* and is usually denoted $\mathbb{R}$. Every real number has a decimal expansion. For rational numbers, the decimal will at some point begin to repeat itself. For example:

$$\frac{1}{3} = 0.33333\ldots = 0.\overline{3} \qquad \frac{1}{7} = 0.\overline{142857}$$

The real numbers

Q: Why the name?

A: To distinguish them from *imaginary* numbers (explained in next Wednesday's lecture)

Fact: there are the *same number* of integers as there are positive whole numbers. In fact, there are the *same number* of rational numbers as there are positive whole numbers. However, there are *more* real numbers than rationals.

The real numbers

The real numbers are *totally ordered*. We can compare any two real numbers and say whether the first one is bigger than the second one, whether the second is bigger than the first, or whether they are equal.

The following are examples of true inequalities:

$$7 < 7.4 < 7.5 \qquad -\pi < -3 \qquad \sqrt{2} < 2 \qquad \sqrt{2} \leqslant 2$$

$$2 \leqslant 2 \qquad -10 < \sqrt{100}$$

Set notation

A set is a collection of objects. If S is a set, we write $a \in S$ to say that a is an element of S. We can also write $a \notin S$ to mean that a is *not* an element of S.

Example: $3 \in \mathbb{Z}$ but $\pi \notin \mathbb{Z}$.

Example of set-builder notation:

$$A = \{1, 2, 3, 4, 5, 6\} = \{x \mid x \in \mathbb{Z} \text{ and } 0 < x < 7\}.$$

Intervals

For $a, b \in \mathbb{R}$,

$$(a, b) = \{\, x \mid a < x < b \,\}$$

whereas

$$[a, b] = \{\, x \mid a \leqslant x \leqslant b \,\}.$$

Now let us look at Table 1 on page A4 of the textbook. This shows how different intervals can be written using interval notation, set-builder notation, and how they can be drawn on the real number line.

Inequalities

Let $a, b, c \in \mathbb{R}$.

1. If $a < b$, then $a + c < b + c$.
2. If $a < b$ and $c < d$, then $a + c < b + d$.
3. If $a < b$ and $c > 0$, then $ac < bc$.
4. If $a < b$ and $c < 0$, then $ac > bc$.
5. If $0 < a < b$, then $\frac{1}{a} > \frac{1}{b}$.

Solving inequalities

Solve the inequality $1 + x < 7x + 5$.

Solving inequalities continued

Find solutions to the following inequalities and write the solutions in interval notation, set-builder notation and indicate the solution on the real line

1. $4 \leqslant 3x - 2 < 13$
2. $x^2 - 5x + 6 \leqslant 0$
3. $x^3 + 3x^2 > 4x$

Absolute value

The absolute value of a number a, denoted by $|a|$ is the distance from a to 0 along the real line. A distance is always positive or equal to 0 so we have

$$|a| \geqslant 0 \text{ for all } a \in \mathbb{R}.$$

Examples

$$|3| = 3 \qquad |-3| = 3 \qquad |0| = 0$$

$$|2-\sqrt{3}| = 2-\sqrt{3} \qquad |3-\pi| = \pi - 3$$

In general we have

$$|a| = a \qquad \text{if } a \geqslant 0$$

$$|a| = -a \qquad \text{if } a < 0$$

Example

Write $|3x - 2|$ without using the absolute value symbol.

$$|3x - 2| = \begin{cases} 3x - 2 & \text{if } 3x - 2 \geqslant 0 \\ -(3x - 2) & \text{if } 3x - 2 < 0 \end{cases}$$

Hence

$$|3x - 2| = \begin{cases} 3x - 2 & \text{if } x \geqslant \frac{2}{3} \\ 2 - 3x & \text{if } x < \frac{2}{3} \end{cases}$$

Properties of Absolute Values

Suppose $a, b \in \mathbb{R}$ and $n \in \mathbb{Z}$. Then

1. $|ab| = |a||b|$
2. $|\frac{a}{b}| = \frac{|a|}{|b|}$ $(b \neq 0)$
3. $|a^n| = |a|^n$

Let $a > 0$. Then

4. $|x| = a$ if and only if $x = a$ or $x = -a$.
5. $|x| < a$ if and only if $-a < x < a$.
6. $|x| > a$ if and only if $x > a$ or $x < -a$.

Example: Solve $|2x - 5| = 3$.

The triangle inequality

If $a, b \in \mathbb{R}$, *then* $|a+b| \leqslant |a| + |b|$.

How do we prove this?

First observe that $-|a| \leqslant a$ and $a \leqslant |a|$.

Similarly, $-|b| \leqslant b$ and $b \leqslant |b|$.

Applying the Triangle Inequality

Example: If $|x - 4| < 0.1$ and $|y - 7| < 0.2$, use the Triangle Inequality to estimate $|(x + y) - 11|$.